HOW FAR CAN A PRAGMATIST GO INTO QUANTUM THEORY? A CRITICAL VIEW OF OUR CURRENT UNDERSTANDING OF QUANTUM PHENOMENA

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To date, quantum mechanics has proven to be our most successful theoretical model. However, it is still surrounded by a "mysterious halo" that can be summarized in a simple but challenging question: Why quantum phenomena are not understood under the same logic as classical ones? Although this is an open question (probably without an answer), from a pragmatist's point of view there is still room enough to further explore the quantum world, marveling ourselves with new physical insights. We just need to look back in the historical evolution of the quantum theory and thoroughly reconsider three key issues: (1) how this has developed since its early stages at a conceptual level, (2) what kind of experiments can be performed at present in a laboratory, and (3) what nonstandard conceptual models are available to extract some extra information. This contribution is aimed at providing some answers (and, perhaps, also raising some issues) to these questions through one of such models, namely Bohmian mechanics, a hydrodynamic formulation of the quantum theory, which is currently trying to open new pathways of understanding.* Specifically, the Chapter constitutes a brief and personal overview on the historic and contextual evolution of this quantum formulation, its physical meaning and interest (leaving aside metaphysical issues), and how it may help to overcome some preconceived paradoxical aspects of the quantum theory.

Keywords: Quantum foundations; quantum hydrodynamics; Bohmian mechanics; hidden variables; quantum coherence; interference; Wheeler's delayed-choice experiment.

1. How Do We Understand the Quantum World?

In 2011, Physics World honored Aephraim Steinberg and colleagues,[1] from the University of Toronto, with the top one position in its yearly breakthrough ranking. So far the quest for fundamental particles involves large energy and length scales — from Giga to Teraelectronvolts (e.g., 7 TeV in p-p collisions), from tenths of meters (e.g., the LHC detector has about 14.6 m of diameter and 21.6 m of total length) to Kilometers (e.g., the LHC ring has about 27 Km length). In contrast, Steinberg's team developed a very interesting top-table experiment[2] at much smaller scales —about 1.3 eV (943 nm) and a few meters—, designed to shed some light on

*It is a difficult task to capture and provide within a few pages feelings and thoughts about the quantum theory accumulated along years of work with analytical developments and numerical simulations. Hence this contribution only intends to be a modest conclusion from such a work, which is, of course, open to debate.

another fundamental aspect of the quantum theory: elucidating how quantum particles travel (on average) through physical space. With the aid of the so-called "weak measurement" technique,[3] consisting of weakly perturbing the quantum system at some time during its evolution and previous to its definite measurement, this team has become "the first to track the average path of single photons passing through a Young's double slit experiment —something that Steinberg says physicists had been "brainwashed" into thinking is impossible." But, why is this experiment relevant at all? The mathematical formulation of the quantum theory is neat and accurate, and its applications have proven to be very powerful —an important "bite" of the gross internal product in industrialized countries relies on quantum mechanics, including developments and applications in technological, energetic or biomedical areas. From the electron to the Higgs, the success of quantum mechanics is indisputable. However, what do we really know about quantum systems? The famous Bohr-Einstein debates[4] ended in the 1930s with the orthodox or Copenhagian view of quantum systems, which has healthfully survived to date: the quantum world is essentially probabilistic and hence it does not make any sense asking questions intended for going beyond probabilities. Actually, these probabilities are such that if we try to determine accurate (probabilistic) information about one of the variables (A) from a pair of (classical) canonically conjugate variables, we will be unable to obtain any relevant information about the other (B), and vice versa. This is the essence of Bohr's complementarity principle, which formally translates into the well-known Heisenberg uncertainty relation,

$$\Delta A \cdot \Delta B \geq \frac{\hbar}{2}, \tag{1}$$

where Δ denotes the dispersion in the measurement of either A or B.

At a more pictorial level, complementarity is often regarded to the alleged wave-corpuscle duality exhibited by quantum systems: an electron behaves as a wave when it passes through a pinhole, and as a corpuscle when it is acted by a highly energetic photon (e.g., γ rays in Compton scattering). Hence, in Young's two-slit experiment, if the electron passes without ever being disturbed during the transit, we will observe a nice fringed pattern: the electron distributes within some regions, while avoids some others. All about the electron distribution is known, but nothing about which slit the electron passed through or, in other words, which momentum the electron carried. Trying to determine the latter would eventually lead to fringe erasure because of the impossibility to measure both at the same time. According to von Neumann,[5] this is a manifestation of a non-unitary evolution process known as the "collapse" of the wave function. That is, while the system is unperturbed, it displays a unitary (probability preserving) evolution in compliance with Schrödinger's equation; once a measurement is performed, such unitarity is broken and the system state randomly "collapses" onto any of the pointer states of the measurement device with the probability prescribed by the system wave function. When this argumentation was proposed, it found the strong opposition of Einstein

and others, who thought that there should be something else, a set of internal or *hidden* variables that would determine the outcome, thus removing any trace of randomness. For those physicists seeking for an objective or realistic description of quantum systems, the wave function could not be complete because it was not able to specifically determine all possible information about the system.

It was a hard and thorny way to disprove von Neumann's statement, according to which quantum mechanics does not admit hidden variables. However, in 1952 David Bohm proposed[6] a counterexample to this theorem, showing that an account of the individual evolution of the system is still compatible with the quantum theory by simply assuming that the wave function acts like a field "guiding" the system (a similar idea was proposed about 25 years earlier by Louis de Broglie[7]). As a consequence, there is no need for a "collapse" postulate —nor even the presence of an external observer. This does not mean that the mathematical structure itself of quantum mechanics has to be changed, but only that there is still much more room for thinking quantum phenomena in different alternative ways. Bohm's suggestion remained almost forgotten until the 1960s, when it called the attention of John Bell.[8] While working at CERN, he decided to go back to von Neumann's theorem and re-examine it with the purpose of systematizing which properties should be satisfied by a quantum hidden-variable theory in order to be valid. In so doing, Bell noticed[9] that what makes quantum mechanics so special is a feature lacking in classical mechanics, namely nonlocality, i.e., the fact that any local disturbance in a quantum system immediately affects the whole system. The most striking example where this property manifests is in quantum entanglement: the correlations between two spatially separate systems that interacted in the far past are very important, since they can be used to transfer quantum information between two distant places without incurring in surperluminal signaling.

Bell's contribution not only started the revolution of quantum technologies, leading to the development of the quantum information theory, quantum computing, quantum cryptography, quantum teleportation, etc., but indirectly he also motivated a reconsideration of Bohm's approach. Nowadays, for some people this is just an alternative quantum theory (this being based on the ontology generated from it); for others it is only an alternative formulation of the quantum theory. If this is simply regarded as a matter of taste, and we decide to remain at a more pragmatic level, what really matters is the fact that Bohmian mechanics has open an alternative pathway to understand quantum systems, justifying Steinberg's statement that our "brainwashed" view of quantum phenomena may be changed by experiments like the one that he and his colleagues performed just on an optical bench.

2. A Single-Event Prescription to Think Quantum Phenomena

2.1. *Single-event experiments*

During the early days of quantum mechanics the weight of statistical mechanics and thermodynamics was too strong —Boltzman's shadow was too long. It is

not strange, therefore, that the Copenhagen interpretation became widely accepted (aided by the neopositivist thought-streams of those years), healthfully surviving up to now. However, at present quantum experiments can be performed in a, by far, more refined way than in the first decades of the XXth century, thus limiting the importance of statistics. More specifically, the system evolution can be monitored in real time (e.g., molecular configurations, entanglement dynamics, electron ionization, surface diffusion, etc.), in contrast with traditional spectroscopic measurements based on the energy domain.[a] Furthermore, since the 1970s we also have interesting interference experiments corroborating that, even if we know absolutely nothing about how each (quantum) particle behaves individually, at least we know that it reaches the detector as a single, localized event, and not as an extended wave. From the former experiments with electrons[10,11] to the latter with large molecular complexes,[12] it has been confirmed that, as time proceeds and the accumulation of (randomly distributed) particle arrivals on a distant detector becomes larger, an incipient interference fringed pattern starts emerging from a seemingly disordered distribution of single, (time) uncorrelated detections, in clear correspondence with the outcome expected from Schrödinger's equation. Statistics thus plays an important role in accounting for the general picture, but says nothing about how each individual detection takes place.

Chapter 2 of Feynman's renowned *Lectures on Physics* starts as[13] "In this chapter we shall tackle immediately the basic element of the mysterious behavior in its most strange form. We choose to examine a phenomenon which is impossible, *absolutely* impossible, to explain in any classical way, and which has in it the heart of quantum mechanics. In reality, it contains the *only* mystery. We cannot make the mystery go away by "explaining" how it works. We will just tell you how it works. In telling you how it works we will have told you about the basic peculiarities of all quantum mechanics." Effectively, the two-slit experiment summarizes the essence ("mystery") of quantum mechanics. However, like Dirac, Feynman also thought that each individual particle self-interfered, apparently being unaware of Pozzi's and Tonomura's experiments on the two-slit experiment. His scientific authority reinforced the Copenhagian viewpoint, but should things have been different if Feynman would have watched Tonomura's movie[11] of his two-slit experiment with electrons? Well, although evidently we have no answer to this question, perhaps we could say that, for someone who succeeded in introducing the concept of trajectory into quantum mechanics (in spite of Bohr's disapproval), things would have been, at least, a bit different.

In any case, it is important to keep in mind three very simple ideas:

i) Appealing to Occam's razor, there is no reason to think that the wave

[a]Even if from a purist's viewpoint this tracking is not exactly in time, the fact is that we can reconstruct a whole "movie" of the system time-evolution, something technically forbidden until recent times.

function collapses to a local point upon detection, as formerly stated by von Neumann.[5]

ii) Experiments corroborate a statistical origin of interference patterns (and, in general, any quantum trait), consisting of a large accumulation of single, localized events.[10–12]

iii) Even if all particles are generated at the same source (where they can interact), once they are released experiments also confirm that each particle arrival is independent of all other previous or subsequent ones (see more recent experiment in[10]). That is, one particle is totally unaware of what others do.

Accordingly, one cannot avoid thinking whether single-event descriptions are affordable in quantum mechanics, even being aware that the evolution of a single particle cannot be tracked (at least, not with the current technology) without irreversibly perturbing it. In some sense, it would still be possible to work at the level of classical fluids: we know nothing about the individual motion of the fluid's constituents, but still its collective dynamical properties could be determined by means of a streamline analysis.

2.2. *Bohmian mechanics*

For simplicity, let us consider the nonrelativistic Schrödinger equation,

$$i\hbar\frac{\partial\Psi}{\partial t} = \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\Psi, \tag{2}$$

which describes how the wave amplitude Ψ associated with a physical system of mass m evolves in time through a given configuration space accounting, for instance, for the system position. Statistical information about the possible outcomes that can be expected at any place and time are determined from the probability density $|\Psi|^2$. It is at this point where Bohm[6] felt the need to introduce the concept of hidden variable, just as a way to test the validity of the assumption that "the most complete possible specification of an individual system is in terms of a wave function that determines only probable results of actual measurement processes." These hidden variables in principle would "determine the precise behavior of an individual system, but which are in practice averaged over in measurements of the types that can now be carried out." The evolution of these hidden variables, which he identifies with individual realizations (trajectories) of the system, arises after recasting Schrödinger's equation as a set of two real equations of motion, one for the probability density ρ and another one for the phase S of the wave function (when the latter is expressed in polar form, $\Psi = \rho^{1/2}e^{iS/\hbar}$). This gives rise to the continuity equation and a Hamilton-Jacobi-like equation, from which Bohm postulates that the system trajectories evolve according to the equation of motion

$$\mathbf{v} = \dot{\mathbf{r}} = \frac{\nabla S}{m}, \tag{3}$$

where $\mathbf{v}$ is a local velocity field. Within this scenario, quantum systems thus consist of a wave field (Ψ) and a particle guided by this field, which follows a trajectory $\mathbf{r}(t)$ in configuration space. This is essentially what is nowadays known as Bohmian mechanics,[14] although the more philosophical (ontological) aspects are or are not seriously considered depending on the author.

Leaving aside metaphysical connotations, from a pragmatist's viewpoint, Bohmian mechanics is basically a fluid-dynamical description for quantum systems. It is worth stressing that the same ideas were already proposed (although not in the context of hidden variables) by de Broglie[15] and Madelung,[16] and contemporarily to Bohm by Takabayasi.[17] Furthermore, it should be clear that this formulation is totally equivalent and is at the same level as any other of the more traditional ones (Schrödinger's, Heisenberg's, Dirac's, Moyal-Wigner, etc.). Each formulation emphasizes a different aspect of the quantum theory; Bohmian mechanics stresses the fact that quantum systems can be associated with a quantum fluid (not in vain Schrödinger's equation is just a diffusion equation with an imaginary diffusion constant, $i\hbar/2m$). Actually, descriptions like the Bohmian one in terms of streamlines are rather common in the literature,[18] and in recent years it has been possible to recreate Bohmian-like systems (in fact, deBroglian ones) by means of classical fluid-dynamical experiments.[19,20]

Rather than hidden variables, Bohmian mechanics constitutes a valuable tool that allows us to determine "hidden" quantum properties, i.e., properties that are not evident within other formulations of the quantum mechanics. For example, based on the Bohmian non-crossing property,[21] i.e., that Bohmian trajectories cannot pass through the same point of the configuration space at the same time, we find that quantum flows do not mix in configuration space, as Steinberg's experiment shows,[2] and accordingly one can properly specify tubes along which the probability remains constant at any time.[22] Also, based on the non-crossing property, one notices that the physical implications of the superposition principle go further beyond from its simple mathematical application due to the phase dynamics involved, which immediately modifies the velocity map throughout the whole configuration space.[21,23] In any case, it is worth stressing that Bohmian trajectories only provide us with hydrodynamical information of the quantum fluid or wave field itself, but not of what is in there behind it —that is, how the average ensemble evolves, but not how a real individual particle moves.[18,23] A simple example of Bohmian trajectories illustrating the renowned Young two-slit experiment is displayed in Fig. 1, where the background contour-plot represents the nonlocal velocity field pervading the whole configuration space.

3. Wheeler's Delayed Choice Experiment Revisited

Let us now revisit the well-known delayed-choice experiment proposed by Wheeler[24] in the late 1970s —a paradigm of the mystery entailed by quantum mechanics— to illustrate how Bohmian mechanics removes the paradoxical aspects introduced

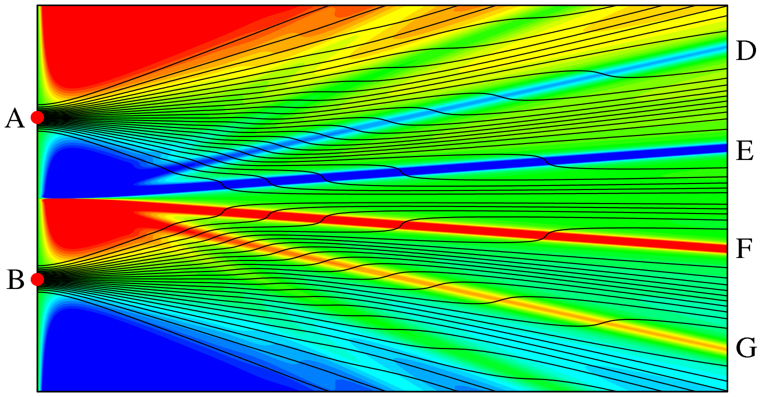

Fig. 1. Numerical simulation of Young's two-slit experiment: the slits centered at A and B, along the x direction, produce two Gaussian wave packets, which eventually superimpose and generate in the long time (Fraunhofer) regime[23] a series of interference fringes (D, E, F, G). Black solid lines represent ensembles of Bohmian trajectories leaving each slit. The background contour-plot corresponds to the velocity field as time proceeds (time increases along the horizontal axis): from blue to red, increasing value of the velocity from negative to positive, respectively. Note that the non-crossing appears as a consequence of the sudden change of sign undergone by the velocity field along the system symmetry line ($x = 0$).

by this experiment. The main idea behind this experiment is very simple: to reveal the puzzling dual wave-corpuscle nature of quantum systems within the same experiment. To this end, Wheeler considers an optical Mach-Zehnder interferometer with a movable second beam splitter. Moreover, the experiment is performed in such a way that at each time there is only one photon inside the interferometer. For a visual representation of the interferometer configurations described below, see Fig. 2 (do not confuse here the Bohmian trajectories displayed with usual optical geometric paths). When the photon enters the interferometer, the first (fixed) beam splitter (BS1), oriented at 45° degrees with respect to the photon incidence direction, may produce direct transmission towards a mirror M1 with a 50% of probability, or a perpendicular deflection (reflection) towards a mirror M2. In either case, when the photon reaches the mirrors, it gets deflected 90° with respect to the corresponding photon incidence direction. Eventually, in an open configuration (see Fig. 2(a)), i.e., without the second beam splitter BS2, an arrival can be registered by a detector D1 if the photon followed the transmitted path (let us denote it by P1), or by D2 if otherwise it followed the reflected path (P2). This is a typical corpuscular scenario, describable in terms of classical optics (the photon follows geometric rays). On the other hand, in a closed configuration (see Fig. 2(b)), when BS2 is introduced at the place where the paths P1 and P2 intersect, the photon will always be detected by D2. In this case the photon displays its wave behavior: at BS2 the components of the associated wave interfere destructively in the direction of P1 and constructively along P2. Following the Bohr-Einstein debates,[4] Wheeler

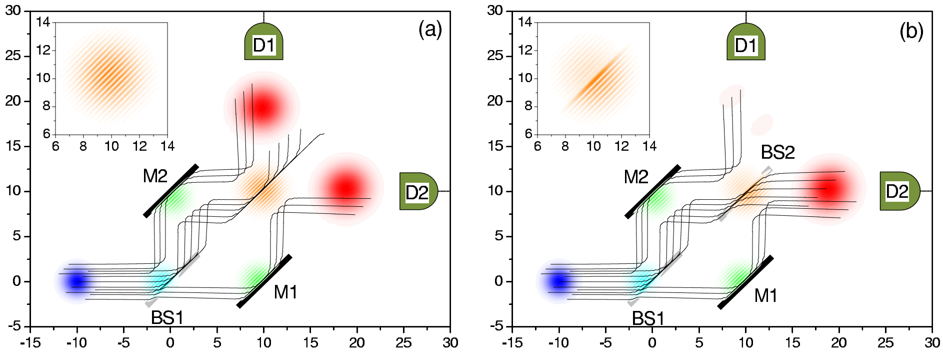

Fig. 2. Numerical simulations[28] of the open (a) and closed (b) interferometer configurations involved in Wheeler's delayed choice. The background monochrome contour-plots correspond to different stages in the evolution of the system wave function inside the interferometer (see text for details): blue: initial state (Gaussian wave packet); light blue: splitting at BS1; green: reflection at the mirrors (M1 and M2); orange: superposition of the two wave packets at the position where BS2 should be allocated; red: final stage (wave packets in their way to the corresponding detectors, D1 and D2). In the insets of each panel, a magnification of the probability density in the region around BS2. The black solid lines represent ensembles of Bohmian trajectories starting with initial positions covering the corresponding regions of the initial probability density.

reformulated one of the main questions coming from them: when does the quantum system make the choice to behave as a wave or as a particle? Instead of considering a double slit, Wheeler assumed the above interferometer, but with BS2 movable, so that it could be removed or inserted once the photon had passed through BS1. The result is very interesting: regardless of when BS2 is put into play, the photon always behaves as it should, that is, just as if it could anticipate what is going to happen in future: the photon makes a delayed choice. Although initially conceived as a thought-experiment, this experiment has already been carried out in the laboratory in many different ways, but always confirming this challenging dual behavior.[25]

The weirdness of Wheeler's experiment readily dissipates if one starts describing it properly, that is, using quantum mechanics since the very beginning, without any aid of classical corpuscle-based model. A priori the usual Schrödinger or Heisenberg descriptions might seem of little help, since they essentially stress the role of probabilities. However, if we keep in mind the fact that there is an associated phase dynamics (and therefore an inherent nonlocal velocity field), things change. This is the idea that first Bohm and coworkers,[26] and later on Hiley and Callaghan,[27] tried to convey by analyzing the experiment in terms of Bohmian mechanics. Because Bohmian trajectories, which in essence are elements to make apparent the flow of such probabilities, obey the non-crossing rule[21] (see Sec. 2.2), what happens is that the photon always behave in the same way. If BS2 is absent, because the trajectories coming from P1 and P2 cannot cross the symmetry line at 45°, those coming from P1 are reflected in the direction of D2, and those from P2 in the direction of D1. That is, it is not that the photon follows P1 or P2 until reaching the corre-

sponding detector, but there is an exchange in the directionality of the associated quantum flow. On the other hand, if BS2 is introduced, even once the photon is inside the interferometer, the wave recombination process taking place at this beam splitter produces that the two sets of trajectories will eventually go into one detector, namely D2. This all-the-way wave behavior (the classical corpuscle notion just disappears, since it is not necessary at all) is illustrated in Fig. 2 by means of the numerical simulation [28] of the open (a) and closed (b) interferometer configurations described above. As it can be seen, there is no choice of the photon at all, but just a modification of the boundary conditions affecting its wave function, which naturally gives rise to different outcomes, regardless of whether BS2 is introduced or removed once the wave function has started its evolution inside the interferometer. This kind of realistic simulations are very important to better understand the physics that is taking place in apparently paradoxical situations, as it has been recently shown in the case of atomic Mach-Zehnder interferometry, [29] for example, which is typically considered to analyze and discuss fundamental complementarity issues due to its suitability to this purpose. [30]

4. Pushing Quantum Mechanics Hard Enough

Quantum mechanics has proven to be the most successful theory ever devised (at least, to date). This theory not only has addressed the most fundamental physical problems, but its applications constitute an important part of our everyday life (actually, more sophisticated applications are still to come). However, the fact quantum phenomena cannot be understood under the same logic as classical ones brings in a puzzling situation, which has left open a tough debate on its interpretation since the 1930s. Probably one may think that this debate will never be really closed (or at least until we will be able to devise a new, more general theory) and that, in such a case, it might be pointless to continue talking about quantum philosophical issues or trying to further develop the area of the quantum foundations. Evidently, this leads to a sort of hopeless situation, with a remarkably close resemblance to Plato's myth of the cave: we are enforced to perceive the shadows cast on the wall of the cave (our reality) by the real world (the Reality), without possibility to ever reaching a true understanding of the physical world.

However, we have seen above that, even within such a harsh scenario, there is still room enough to further explore the quantum world from a pragmatist's point of view, just playing around with the quantum rules and its many way to formulate them. One only needs to look back for a while and make a reflection on how the quantum theory has conceptually developed since its early stages, what can be done at present, and which alternative routes can be followed. These are the essential ingredients for new quantum developments and advances. Based on recent achievements, the Bohmian formulation of the quantum theory seems to be one of these routes, which allows us to understand how quantum systems evolve obeying a non-observable (i.e., not directly accessible in the experiment) phase dynamics.

Perhaps it will not be possible to determine how a real individual particle evolves without disturbing it, but at least now we have a tool to understand how the flow of many of these identical particles evolves in configuration space. This has helped us to understand that there are no paradoxes in the quantum theory, but only a misconception about how quantum systems behave, anchored in old-fashioned classical prejudices. In this regard, following Nobel Laureate Anthony Leggett,[31] "if we push quantum mechanics hard enough it will break down and something else will take over —something we can't envisage at the moment."

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